LCR

*The London Close Reasoner*

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# Syllogistic Logic

THIS NOTE SUMMARIZES some key points about syllogistic logic.

## 1. TERMS

We first divide *terms* into two species: *singular* terms and *general* terms. Singular terms refer to single individual objects; *e.g.* "Brian Mulroney", "the Pope", "Germany". General terms stand for concepts; *e.g.* "politician", "religious leader", "country".

General terms may be further divided into two species: *monadic* general terms and *polyadic* general terms. (The latter are sometimes called 'relation terms'.) The basis of the division is the number of singular terms required to convert the general term into a proposition, that is, a singular proposition, one constructed by combining general and singular terms. "Rich" is monadic, since it only requires one general term; *e.g.* "Mary is rich". "Taller than" is polyadic (in particular, dyadic) since it requires two singular terms; *e.g.* "Mary is taller than John".

Syllogistic logic is based on monadic general terms, and in the rest of this ***Note***, that is the only kind of general term that will be considered.

### *1.1 Symbolizing terms*

In syllogistic logic, as in deductive logic in general, we concentrate on the pure forms of the statements and arguments. To do this, we go in for massive abbreviation, usually using single letters for the terms. It is conventional to use capital letters for the general terms and lower case letters for the singular terms. Thus, we represent "Mary is rich" as "Rm".

Of course, it is necessary to say which letter abbreviates which term. In the case of singular terms, we can just write down an 'equation':

m = Mary

In the case of general terms, the best way is *the x-method*, which looks like this:

Rx = x is rich

This tells us that "R" abbreviates the general term "rich", and does so by indicating how the general term can be converted to a singular proposition by adding a singular term; the 'x' marks the place where you would put the singular term if you were going to construct such a proposition. This method is especially useful in complex cases, as

Tx = Mary is taller than x

which gives us an abbreviation for the rather awkward general term, "something than which Mary is taller".

### *1.2 Universe of discourse*

It is often convenient to specify a Universe of Discourse. This is a broad class of objects that includes all the referents of all the concepts you mean to employ in a particular problem. If you don't specify a Universe of Discourse you will be taken to be considering the broadest universe which includes absolutely all the objects that there are. This is sometimes appropriate, but often it makes things easier to specify a narrower universe, such as the universe of persons, of countries, of books, or whatever.

### *1.3 Complement terms*

Terms come in pairs; for every term there is another term, called the complement term, which is true of all the objects in the Universe of Discourse that the first term is *not* true of. In English one can form complement terms with the use of a negative prefix, such as 'non'; in the universe of persons, for example, some are rich, and the rest fall under the complement term, and are non-rich. (What about people who are in between, being neither rich nor non-rich? There are no such people; in this part of logic we assume that all our concepts are sharply defined, and everybody falls on one side of the line or the other.)

When using letters to abbreviate terms, one can use a bar above the letter to represent the complement, or one can use an apostrophe, as will be done in this ***Note***. (The bar is better, but the ***LCR*** word processor can't handle it.)

The rule of double negation applies to complements; the complement of the complement is the same as the original term. The non-non-rich are just the rich.

## 2. Categorical Propositions

A categorical proposition is made by combining two monadic general terms, for example, Sailors and Pirates in one of the following four forms, which are labelled by the first four vowels of the alphabet:

A: All sailors are pirates.
E: No sailors are pirates.
I: Some sailors are pirates.
O: Some sailors are not pirates.

In these propositions, the first term, occupying the position of "sailors" is known as the *subject* term; the other term, in the "pirates" position, is known as the *predicate* term.

Every categorical proposition is said to have a *quantity*, either *universal* or *particular*. The A and E propositions are universal; the I and O are particular.

Every categorical proposition is also said to have a *quality*, either *affirmative* or *negative*. The A and I propositions are affirmative; the E and O are negative. (This helps some people remember the letters for the proposition since they can relate the A and I to the Latin *affirmo* (I affirm) and the E and O to *nego* (I deny).)

## 3. Square of Opposition

Categorical propositions having the same subject and predicate terms stand in important logical relationships to each other, and it is customary to show these in a diagram called the Square of Opposition.

### *3.1 Contradictories*

The most important of these relationships is that of *contradiction*. To say that one proposition contradicts another, or that the two are contradictories of each other, is to say that one of them must be false and the other true.

The A and O are contradictories; if it is true that all sailors are pirates, then it is false that some sailors are not pirates, and if it is true that some are not then it is false that all are.

Likewise, the E and I are contradictories. If it is true that no sailors are pirates then it is false that some sailors are pirates, and if some are then it is false that none are.

### *3.2 The Two Interpretations*

The other relationships in the Square of Opposition depend on which interpretation is made of the categorical propositions. No matter what the interpretation, the contradiction relationships are as stated above, and further, the particular propositions are understood in the sense of "at least one". That is, "Some sailors are pirates" is understood to be true if and only if there is at least one sailor who is a pirate. "Some sailors are not pirates" is understood to be true if and only if there is at least one sailor who is not a pirate.

If we combine the contradiction relationships and the "at least one" understanding together, we get the curious result that if in fact there are no sailors at all in the universe of discourse, then particular propositions about sailors will all be false, and so the contradictory universal propositions about them will all be true. It will be true that all sailors are pirates and also true that no sailors are pirates!

This may sound strange, but it is the line taken by modern logic, and has been, on the whole, since the latter part of the nineteenth century. We call it the *modern* interpretation.

But the main logical tradition from Aristotle down to the nineteenth century took a different line; it rejected the idea that a universal proposition is true when the subject term is empty by not applying the system to the case of empty terms. That is, it made the assumption that none of the terms in categorical propositions are empty, neither the terms themselves nor their complements. If there are no sailors, then don't talk about sailors with categorical propositions. We call this the *traditional* interpretation.

Most of the time it doesn't matter which interpretation you use, but sometimes it does make a difference, and then you must choose whichever interpretation is appropriate to the case in hand.

### *3.3 The Traditional Square*

On the modern interpretation, the only relationships there are the contradictory relations on the diagonals. But on the traditional interpretation there are additional relationships around the sides. The A and O are *contraries;* that is, they cannot both be true, though they could both be false. The I and O are *subcontraries;* that is, they cannot both be false, though they could both be true. The I and O are *subalterns* of the A and E respectively; that is, if the A is true the I must be true, and if the E is true the O must be true. The traditional square is shown in figure 1; it is based on that of William of Sherwood, the 13th century English logician whose Tree of Porphyry is considered in another ***Note***. [*Introduction to Logic*, translated by Normal Kretzmann (Minneapolis: University of Minnesota Press, 1966).]

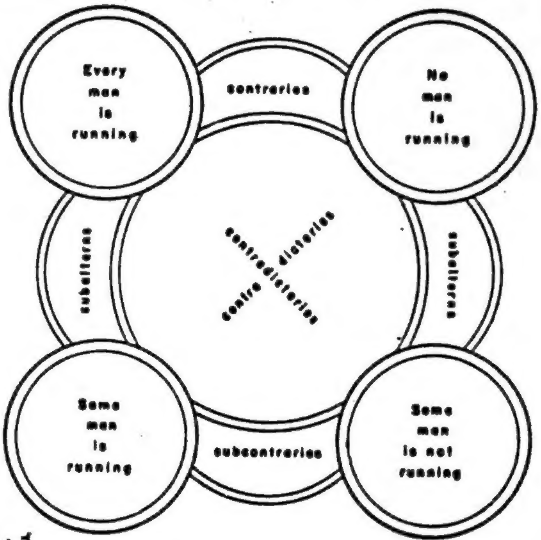


Fig 1

## 4. Immediate Inferences

There are certain relationships among categorical propositions knows as 'immediate inferences'.

### *4.1 Conversion*

The *converse* of a categorical proposition is the same proposition with the terms reversed. For example, **All P is S** is the converse of **All S is P.** The E and I propositions are logically equivalent to their converses. The A and O propositions, by contrast, are *not* equivalent to their converses.

### *4.2 Contraposition*

The *contrapositive* of a categorical proposition is made by switching the order of the terms, and taking their complements. Thus **All non-P is non-S** is the contrapositive of **All S are P.** The A and O propositions are logically equivalent to their contrapositives; the E and I are not. Contraposition applies just in the cases in which conversion does not.

### *4.3 Obversion*

The *obverse* of a categorical proposition is made by changing the quality of the proposition (from affirmative to negative or from negative to affirmative) and replacing the predicate by its complement. Thus **No S are non-P** is the obverse of **All S are P.** All the categorical propositions are logically equivalent to their obverses.

### *4.4 Per Accidens*

All of the above equivalences hold on both the modern and the traditional interpretation. But on the traditional interpretation there are two further relationships top consider. These are not equivalences, but just logical entailments; they only go one way.

One of these is *conversion per accidens* (sometimes called conversion by limitation.) You can go from **All S are P** to **Some P are S,** though of course not *vice versa.* This is the same as going from an A to an I by subalternation and then applying conversion to the I. And subalternation, we recall, only holds on the traditional interpretation.

The other is *contraposition per accidens* (or by limitation). You can go from **No S are P** to **Some non-P are not non-S.** This amounts to going from the E to the O by subalternation, and then taking the contrapositive of the O. Again, this only works on the traditional interpretation.

Conversion *per accidens* has a certain utility in traditional logic, as we shall see, but it is a rare day when you are called upon to use contraposition *per accidens.*

## 5. Syllogism

A syllogism is an argument made of three categorical propositions with exactly three terms arranged in such a way that the subject of the conclusion appears in one of the premises and the predicate of the conclusion appears in the other.

### *5.1 Parts of a Syllogism*

The predicate of the conclusion is called the *major term* and the subject of the conclusion is called the *minor term.* The term which doesn't appear in the conclusion is called the *middle term.* The premise containing the major term is the *major premise;* the premise with the minor term is the *minor premise.*

### *5.2 Standard form*

Though it makes no difference to the logical validity of a syllogism, it is customary and convenient to arrange a syllogism so that the major premise comes first. When that is done, the syllogism is said to be in *standard form.*

### *5.3 Mood and Figure*

When syllogisms are put in standard form, their logical forms can be identified by two attributes known as mood and figure.

The *mood* of a syllogism identifies the major and minor premise and the conclusion each as being A, E, I or O propositions. For example, if the major premise is an A proposition, the minor premise an E proposition and the conclusion an O proposition, then the mood is said to be AEO. This means that there are just 4x4x4=64 possible moods.

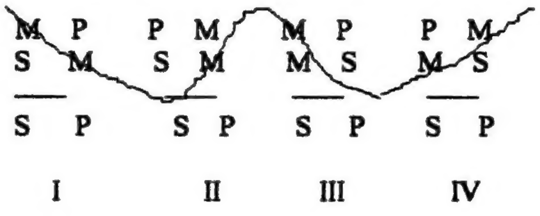
The *figure* of a syllogism identifies the arrangement of the terms in the premises. There are just four possible arrangements called figures I, II, III and IV.

The figures are defined by the placement of the middle term:

Figure I: Middle term is subject of the major premise and predicate of the minor premise.
Figure II: Middle term is predicate of both premises.
Figure III: Middle term is subject of both premises.
Figure IV: Middle term is predicate of the major premise and subject of the minor premise.

There are thus 4 x 64 = 256 possible syllogistic forms. Modern logic recognizes just fifteen of these as valid. Traditional logic adds another nine to make twenty-four. The chances are thus less than one in ten that a syllogism composed at random will be valid.

One way to remember the figures to use the following diagram:



(Draw a line through the middle terms and think 'W'.)

### *5.4 The Fourth Figure Issue*

If you don't require your syllogisms to be put in standard form, then you won't care what order the premises are in, and so you won't see any difference between Figure I and Figure IV, and you will only count three figures; you will regard Figure IV syllogisms as just Figure I syllogisms with their premises in an unusual order. That is the way Aristotle did it, and that view prevailed up through the Middle Ages, and even beyond; Isaac Watts, for example, in the 18th century was a three figure man. Richard Whately, however, in the 19th century was for four.

It is not known when and by whom the fourth figure was introduced. The Arabic philosopher Averroes [a-ver'-o-eez] (Ibn Ruoushd) who lived in Spain and Morocco (1126-98) knew of it and mistakenly attributed it to the ancient physician and philosopher Galen (130-201 AD); Galen, however, believed in only three.

## 6. Testing for Validity

There are many different ways of determining whether or not a syllogism is valid. Several are presented here.

### *6.1 The Code Names*

There are only twenty-four valid syllogistic forms on the traditional interpretation (only fifteen on the modern) so it is not impossible simply to memorize them! This is not as hard as it seems.

Actually, you only have to learn the nineteen which form the basis for the traditional system. It is quite easy then to handle the four that are not recognized on the modern view, and to add the further five, called subaltern moods, that are recognized on the traditional.

This can be done by using certain code names for the syllogistic forms that were worked out, probably in the 13th century. William of Sherwood's logic book is the earliest surviving instance of the use of these words, but it seems clear that he got it from earlier logicians.

The main thing about the code names is that the first three vowels of the name give the mood of the syllogism. The names are then grouped according to figure in an easily remembered verse. It is given below with the syllogism names in capital letters; the other words give the figure in Latin. Underlines are used to indicate accent, and so bring out the poetry of it. [This is the accenting of Otto Bird in *Syllogistic and its Extensions* (Prentice-Hall, 1964), p. 23; some Latinists would do it differently.] The figures are indicated on the left.

I. BARBARA, CELARENT, — DARII,
FERIO-que prioris.
II. CESARE, CAMESTRES, — FESTINO,
BAROCO secundae.
III. Tertia DARAPTI, — DISAMIS, DATISI,
FELAPTON,
BOCARDO, FERISON habet. — Quarta
insuper addit
IV. BRAMANTIP, CAMENES, DIMARIS,
FESAPO, FRESISON.

For the modern interpretation, you drop all the ones that have two universal premises and a particular conclusion; these are Darapti, Felapton, Bramantip and Fesapo. The subaltern moods, which the traditional interpretation adds, are syllogisms in which you draw a particular conclusion when you could have drawn a universal one. You have such moods based on Barbara, Celarent, Cesare, Camestres and Camenes.

Most of the consonants in these code names also have a meaning, which we will go into elsewhere.

If you know this verse, then you can put your syllogism into standard form and then recite the verse to yourself until you get to the figure your syllogism has, and then look out for the mood.

Very few people in the present day, however, know this verse or these code names; most logic books do not even mention them. Knowing them, therefore, is a good way of being one up on the competition.

## *6.2 The Rules*

If a syllogism is going to be valid it has to conform to certain rules. Knowing and applying these rules is one of the more common ways of dealing with syllogisms. Most logic books that cover syllogism contain some version of these rules.

The rules make use of a special technical concept, *distribution*, and it is necessary to master that before learning the rules.

The key idea to grasp is this: A categorical proposition distributes a term if and only if what the proposition says about the term it also says about every subclass of the term.

For example, the A proposition distributes its subject term. **All sailors are pirates** says about the term **sailors** that they are all pirates, and it also says this about every subclass of sailors; for example, it says that **all fat sailors are pirates.**

On the other hand, the A proposition does not distribute its predicate term. What it says about the predicate does not extend to the subclasses. **All sailors are pirates,** for example, does not say that **all sailors are fat pirates.**

If you work this through for all four of the A, E, I and O propositions you will reach the conclusion that Universal propositions distribute their Subject terms and that Negative propositions distribute their Predicate terms. It is possible to remember this with the aid of the motto, "Under Study, Never Pass." (Some Americans remember it with "Uncle Sam Never Panics," but this does not embody a vital truth, as does our motto.)

The rules may now be set out as follows; this set has more rules than you strictly need, but the extra ones are useful in bringing out the symmetry of the situation.

### THE RULES

RULES OF DISTRIBUTION

1. The Middle Term must be distributed at least once.
2. A term distributed in the conclusion must be distributed in the premises.

RULES OF QUALITY

3. You need an affirmative premise.
4. A negative premise requires a negative conclusion.
5. A negative conclusion requires a negative premise.

RULES OF QUANTITY

6. You need a universal premise.
7. A particular premise requires a particular conclusion.

[8. A particular conclusion requires a particular premise.] (This rule is for the modern interpretation only.)

A syllogism which obeys all these rules is valid; if it breaks one it is invalid. Notice that the addition of Rule 8 is the only difference between the modern and traditional interpretations.

## *6.3 Diagrams*

It is very common to use diagrams for checking the validity of syllogisms. The diagram system now in widest use is that of John Venn (1881). These work best for the modern interpretation, but can handle the traditional. An earlier system of circles is usually credited to the mathematician Leonhard Euler (1761). Euler circles assume the traditional interpretation, and are rather awkward to use. One sees them occasionally nowadays, though not often.

A third system uses lines rather than circles and originated with philosopher and mathematician Johann Heinrich Lambert (1764). It is very likely that the ***LCR*** will be the only place to give the Lambert lines system in the present day. It is limited to the traditional interpretation, but is perhaps the easiest to use for that purpose.

Venn Circles and Lambert Lines (and perhaps also Euler Circles) will be discussed in other ***Notes***.

***The London Close Reasoner***

Prof. Robert W Binkley
LCR
Department of Philosophy
University of Western Ontario
London, Ont. N6A 3K7 Canada

Talbot College 324
(519)-661-3453 Ext. 3750